CIMT Further Statistics p67 Ex4B.

1. X = mass of crisps in bag

$X \sim N(\mu, 0.5^2)$

X_1 = mass of crisps before overhaul

$X_1 \sim N(\mu_1, 0.5^2)$

$n_1 = 6$

X_2 = mass of crisps after overhaul

$X_2 \sim N(\mu_2, 0.5^2)$

$n_2 = 12$

$H_0: \mu_1 = \mu_2$ (ie overhaul has had no effect)

$H_1: \mu_1 \neq \mu_2$

Assume H_0 to be true

$\alpha = 5\%$

two tail test

from samples, $\bar{x}_1 = 151.8$ and $\bar{x}_2 = 150.592$.

we have $X_1 \sim N(\mu_1, 0.5^2)$ $\quad X_2 \sim N(\mu_2, 0.5^2)$

$\bar{X}_1 \sim N\left(\mu_1, \frac{0.5^2}{6}\right)$ $\quad \bar{X}_2 \sim N\left(\mu_2, \frac{0.5^2}{12}\right)$

$\Rightarrow \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{0.5^2}{6} + \frac{0.5^2}{12}\right)$

so $\dfrac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{0.5^2}{6} + \frac{0.5^2}{12}}} \sim N(0, 1^2)$

as $\mu_1 = \mu_2 \Rightarrow \mu_1 - \mu_2 = 0$

so test statistic, $Z = \dfrac{(151.8 - 150.592) - (0)}{\sqrt{\frac{0.5^2}{6} + \frac{0.5^2}{12}}}$

$= 4.83333$

$P(Z > 4.8333) = 0.0000006722$

p-value $= 2 \times P(Z > 4.8333)$

$= 0.000001$

< 0.05

So we have evidence to reject H_0 and conclude that there is a difference in the mean mass of crisps since the overhaul of the machine.

We conjecture that the overhaul has lowered the mean mass of crisps.

2.

X = difference in breaking strengths.

$X \sim N(\mu, 5^2)$

let $X_A \sim N(\mu_A, 5^2)$ $\quad$ $X_B \sim N(\mu_B, 5^2)$

$n_A = 9$ $\quad$ $n_B = 8$

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$.

Assume H_0 to be true

$\alpha = 5\%$.

two tail test

from calculation, $\bar{x}_A = 75.8222$ $\quad$ $\bar{x}_B = 78.7375$

$$X_A \sim N(\mu_A, 5^2) \qquad X_B \sim N(\mu_B, 5^2)$$

$$\bar{X}_A \sim N\left(\mu_A, \frac{5^2}{9}\right) \qquad \bar{X}_B \sim N\left(\mu_B, \frac{5^2}{8}\right)$$

so $\bar{X}_A - \bar{X}_B \sim N\left(\mu_A - \mu_B, \frac{5^2}{9} + \frac{5^2}{8}\right)$

$$\frac{\bar{X}_A - \bar{X}_B - (\mu_A - \mu_B)}{\sqrt{\frac{5^2}{9} + \frac{5^2}{8}}} \sim N(0, 1^2)$$

test statistic, z, $= \dfrac{(75.8222 - 78.7375) - (0)}{\sqrt{\frac{5^2}{9} + \frac{5^2}{8}}}$

$= -1.19992$

so P-value $= 2P(Z < -1.19992)$

$= 2 \times 0.115086$

$= 0.230171$

> 0.05

so we donot have evidence to reject H_0 and therefore conclude that the mean breaking strengths are the same from each manufacturer.

3. X = no. crates completed per hour.

$X \sim N(\mu, \sigma^2)$

let $X_1 \sim N(\mu_1, 10)$ $\quad$ $X_2 \sim N(\mu_2, 25)$

$n_1 = 10$ $\quad$ $n_2 = 10$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Assume H_0 to be true

$\alpha = 5\%$

Two tail test

from calculation, we have $\bar{x}_1 = 82.4$ $\quad$ $\bar{x}_2 = 78$

$X_1 \sim N(\mu_1, 10)$ $\quad$ $X_2 \sim N(\mu_2, 25)$

$\bar{X}_1 \sim N(\mu, \frac{10}{10})$ $\quad$ $\bar{X}_2 \sim N(\mu_2, \frac{25}{10})$

$\bar{X}_1 \sim N(\mu, 1)$ $\quad$ $\bar{X}_2 \sim N(\mu_2, 2.5)$

so $\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, 1 + 2.5)$

$\bar{X}_1 - \bar{X}_2 \sim N(\mu_1 - \mu_2, 3.5)$

so $\dfrac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{3.5}} \sim N(0, 1^2)$

test statistic, $z = \dfrac{82.4 - 78 - (0)}{\sqrt{3.5}}$

$= 2.3519$

p-value $= 2 \times P(Z > 2.3519)$

$= 2 \times 0.009339$

$= 0.018678$

< 0.05

Hence, we have evidence to reject H_0 and conclude that the mean number of crates completed per hour have different means for the two production lines.

We conjecture that the new line has a higher mean number of crates completed per hour.

4. X_A = mass filled by machine A $\quad$ X_B = mass filled by machine B

$X_A \sim N(\mu_A, 5^2)$ $\quad$ $X_B \sim N(\mu_B, 3^2)$

$n_A = 12$ $\quad$ $n_B = 15$

$H_0: \mu_A = \mu_B$

$H_1: \mu_A \neq \mu_B$.

Assume H_0 to be true.

$\alpha = 5\%$

two tailed test.

we also have $\bar{x}_A = 301.75$ $\quad$ $\bar{x}_B = 299$

so $X_A \sim N(\mu_A, 5^2)$ $\quad$ $X_B \sim N(\mu_B, 3^2)$

$\bar{X}_A \sim N(\mu_A, \frac{5^2}{12})$ $\quad$ $\bar{X}_B \sim N(\mu_B, \frac{3^2}{15})$

$$\bar{X}_A - \bar{X}_B \sim N\left(\mu_A - \mu_B, \frac{5^2}{12} + \frac{3^2}{15}\right)$$

so
$$\frac{\bar{X}_A - \bar{X}_B - (\mu_A - \mu_B)}{\sqrt{\frac{5^2}{12} + \frac{3^2}{15}}} \sim N(0, 1^2)$$

test statistic,
$$z = \frac{(301.75 - 299) - (0)}{\sqrt{\frac{5^2}{12} + \frac{3^2}{15}}}$$

$= 1.67879$

so p-value $= 2 \times P(Z > 1.67879)$

$= 2 \times 0.046597$

$= 0.093194$

> 0.05

Hence we do not have sufficient evidence to reject H_0 and conclude that the two machines have the same mean mass filled.

5

a) X_1 = time for technician to repair

$X_1 \sim N(\mu_1, 80^2)$

$n_1 = 12$

X_2 = time for trainee to repair

$X_2 \sim N(\mu_2, 40^2)$

$n_2 = 14$

$H_0: \mu_A = \mu_B$

$H_1: \mu_A \neq \mu_B$.

Assume H_0 to be true

$\alpha = 5\%$ two tail test

we have $\bar{x}_1 = 274.25$ $\bar{x}_2 = 296.571$

test statistic, $z = \dfrac{274.25 - 296.571 - (0)}{\sqrt{\frac{80^2}{12} + \frac{40^2}{14}}}$

$z = -0.877126$

p-value $= 2 \times P(Z < -0.877126)$
$= 2 \times 0.190209$
$= 0.380418$
> 0.05

So we have insufficient evidence to reject H_0 and so we conclude that there is no difference in the mean times of repairing.

b) if the trainee's times are all now increased by 30 mins, this will make $\bar{x}_2 = 326.571$

This will most likely lead to the test statistic being inside the critical region, and the original null hypothesis being rejected.

By quick re-calculation on the Nspire, we get $z = -2.05598$ with a p-value of 0.039784 and so, yes, the H_0 would be rejected. This means that the mean times for repair are no longer equal.

6.

a) X_J = times for James' rounds $\quad$ X_A = times for Alison's rounds

$X_J \sim N(\mu_J, 5^2)$ $\quad$ $X_A \sim N(\mu_A, 5^2)$

$n_J = 12$ $\quad$ $n_A = 11$

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A < \mu_B$

We assume H_0 to be true

$\alpha = 5\%$

one tail test

we have that $\bar{x}_J = 37$ $\quad$ $\bar{x}_A = 41$

test statistic, $z = \dfrac{(37-41)-(0)}{\sqrt{\frac{5^2}{12} + \frac{5^2}{11}}}$

$z = -1.91652$

p-value $= P(Z < -1.91652)$

$= 0.027649$

< 0.05

So we conclude that we have sufficient evidence to reject H_0 and conclude that Alison's claim that her mean time is longer is supported by the data.

b) If, instead, the H_1 had been $\mu_A \neq \mu_B$, we would perform a two-tailed test, whose p-value would be $2 \times 0.027649 = 0.055299 > 0.05$ (just!)

This would result in us coming to a different conclusion, in that the mean times could not be argued to be different.