CIMT Further Statistics p51 Ex 3D

1. we have four $5\times5\,m^2$ samples, with 2 small faults.

X = no. faults per $100\,m^2$

$X \sim Po(\lambda)$

$H_0: \lambda = 4\times0.925 = 3.7$

$H_1: \lambda < 3.7$

Assume H_0 to be true.

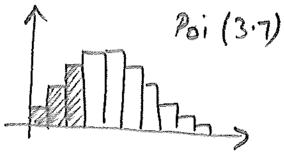
$\alpha = 5\%$

One-tail test.

p-value $= P(X \leq 2)$

$= 0.285433$

> 0.05



so we do not have evidence to reject H_0 and so we conclude that the mean number of faults has <u>not</u> been reduced from 0.925 per $25\,m^2$.

2. 8 parasites found on 3 fish

X = total number of parasites on 3 fish.

$X \sim Po(\lambda)$

$H_0: \lambda = 3 \times 4 = 12$

$H_1: \lambda \neq 12$

Assume H_0 is true

$\alpha = 5\%$

two tail test

as poisson distribution is not symmetrical, we need to be careful about calculating p-value.

$P(X \leq 8) = 0.155028$ from poissCdf (12, 0, 8)

This is clearly inexcess of 2.5%, and 5%, so we can confidently conclude that we cannot reject H_0 and thus it is reasonable to conclude that there are 4 parasites per fish.

3. X = no. of complaint letters in 150 days.

$X \sim Po(\lambda)$

$H_0: \lambda = 150 \times 3 = 450$

$H_1: \lambda < 450$

we assume H_0 to be true.

$\alpha = 1\%$

one-tail test

so $X \sim Po(450)$

$P(X \leq 407) = 0.021263$

< 0.05

OR

we can approx with normal, as $\lambda > 10$

let Y = normal approx to X

$Y \sim N(450, 450)$

$P(X \leq 407) = P(Y < 407.5)$ by c.c.

$= P\left(Z < \frac{407.5 - 450}{\sqrt{450}}\right)$

$= P(Z < -2.00347)$

$= 0.022563$

< 0.05

either way, we have evidence to reject H_0 and conclude that there are fewer than 3 letters of complaint per day.

Assumption needed is that the number of letters per day is independent of other days ie. there has not been a single incident that triggered a lot of complaints, for example.

4. X = no. of breakages per 8 weeks.

$X \sim Po(\lambda)$

$H_0: \lambda = 27 \times 8 = 216$

$H_1: \lambda < 216$

Assume H_0 to be true

$\alpha = 5\%$

one-tail test

we observed a total of $23+19+\ldots+22+26 = 184$ breakages

$P(X \leq 184) = 0.014364$ from poissCdf $(216, 0, 184)$

< 0.05

OR

let Y be normal approx to X as $\lambda > 10$

$Y \sim N(216, 216)$

$P(X \leq 184) = P(Y < 184.5)$ by c.c.

$= P\left(Z < \frac{184.5 - 216}{\sqrt{216}}\right)$

$= P(Z < -2.1433)$

$= 0.016044$

< 0.05

either way, we will reject H_0 and conclude that the managers' announcement has been heeded, resulting in the mean no. of breakages per week to be less than 27.