CIMT Further Stats p75 Ex 4D

1. X = difference in scores. (difference = theory − practical)

assume X to be normally distributed (as given)

$X \sim N(\mu, \sigma^2)$

Student	A	B	C	D	E	F	G	H	I	J	K.
Theory	30	42	49	50	63	38	43	36	54	42	26
Practical	52	58	42	67	94	68	22	34	55	48	17
T − P	−22	−16	7	−17	−31	−30	21	2	−1	−6	9

$H_0: \mu = 0$

$H_1: \mu \neq 0$

Assume H_0 to be true

$\alpha = 5\%$

two tail test

we estimate σ from (T−P)'s $S_{n-1} = 16{\cdot}9131$ and we have $n = 11$ (small) so we perform a t-test on the differences. (so turning this into a single sample t-test)

we also know that $\bar{x} = -7{\cdot}63636$

so $X \sim N(0, \sigma^2)$

$\bar{X} \sim N\left(0, \frac{\sigma^2}{11}\right)$ where $\bar{X}$ = mean difference

so $\dfrac{\bar{X} - 0}{\sqrt{\frac{S^2}{11}}} \sim t_{10}$ as we estimate σ with S_{n-1}

$$p\text{-value} = 2 \times P\left(t_{10} < \frac{-7{\cdot}63636 - 0}{\sqrt{\frac{16{\cdot}9131^2}{11}}}\right)$$

$$= 2 \times P(t_{10} < -1{\cdot}49747)$$

$$= 2 \times 0{\cdot}082576$$

$$= 0{\cdot}165153$$

$$> 0{\cdot}05$$

No evidence to reject H_0, and so we conclude that the mean difference in scores is zero, which suggests that the scores on the Theory and Practical papers are similar.

2. X = differences in sales figures. (where difference = after − before)

Assume X to be normally distributed

let $X \sim N(\mu, \sigma^2)$

Region	A	B	C	D	E	F	G	H	I	J	K
Before	2.4	2.6	3.9	2.0	3.2	2.2	3.3	2.1	3.1	2.2	2.8
After	3.0	2.5	4.0	4.1	4.8	2.0	3.4	4.0	3.3	4.2	3.9
A−B	0.6	−0.1	0.1	2.1	1.6	−0.2	0.1	1.9	0.2	2	1.1

$H_0: \mu = 0$

$H_1: \mu > 0$

Assume H_0 to be true

$\alpha = 5\%$

one tail test

by looking at "After − Before" we create a single sample of size $n = 11$

we estimate σ from $s_{n-1} = 0.907043$, and we have small n, so we perform a single sample t-test on the differences.

we also know that $\bar{x} = 0.854545$.

so $X \sim N(0, \sigma^2)$

$\bar{X} \sim N\left(0, \frac{\sigma^2}{11}\right)$ where $\bar{X}$ = mean difference

$\frac{\bar{X} - 0}{\sqrt{\frac{s^2}{11}}} \sim t_{10}$ as we estimate σ with s_{n-1}

$$\text{p-value} = P\left(t_{10} > \frac{0.854545 - 0}{\sqrt{\frac{0.907043^2}{11}}}\right)$$

$$= P(t_{10} > 3.12467)$$

$$= 0.005394$$

$$< 0.05$$

Hence we have evidence to reject H_0 and so the mean difference in sales is greater than zero, which tells us that sales have increased after the campaign.

3. $X =$ difference in accuracy. (where difference = crackshot − fastfire)

Assume X is normally distributed

$X \sim N(\mu, \sigma^2)$

$H_0 : \mu = 0$

$H_1 : \mu > 0$

Competitor	A	B	C	D	E	F	G	H	I	J
Crackshot	93	99	90	86	85	94	87	91	96	79
Fastfire	87	91	86	87	78	95	89	84	88	74
C−F	6	8	4	−1	7	−1	−2	7	8	5

Assume H_0 to be true

$\alpha = 5\%$

one tail test

we calculate "C−F" to give a single sample of size 10

we estimate σ to be $s_{n-1} = 3.95671$, and thus we do a single sample t-test

we also know $\bar{x} = 4.1$

so $X \sim N(0, \sigma^2)$

$\bar{X} \sim N\left(0, \frac{\sigma^2}{10}\right)$ where $\bar{X}$ = mean difference in accuracy.

$\frac{\bar{x} - 0}{\sqrt{\frac{s^2}{10}}} \sim t_9$ as we estimate σ with s_{n-1}

$$p\text{-value} = P\left(t_9 > \frac{4.1 - 0}{\sqrt{\frac{3.95671^2}{10}}}\right)$$

$= P(t_9 > 3.2768)$

$= 0.004789$

< 0.05

Hence we have evidence to reject H_0 and conclude that the mean difference in accuracy is greater than zero. This suggests that Crackshot shotguns are more accurate.

4. X = difference in scores (where difference = third - fourth)

Assume X to be normally distributed

so $X \sim N(\mu, \sigma^2)$

Competitor	A	B	C	D	E
Third	76	75	72	75	79
Fourth	70	73	71	68	76
T-F	6	2	1	7	3

$H_0: \mu = 0$ (no diffence in scores)

$H_1: \mu \neq 0$ (one round is better than the other)

Assume H_0 is true

$\alpha = 5\%$

two tail test

we calculated '3rd round - 4th round' to give a single sample of size $n=5$

we estimate σ with $s_{n-1} = 2.58844$, and we have small n, so perform a single sample t-test

we also know $\bar{x} = 3.8$

so $X \sim N(0, \sigma^2)$

$\bar{X} \sim N(0, \frac{\sigma^2}{5})$

$\frac{\bar{X} - 0}{\sqrt{\frac{s^2}{5}}} \sim t_4$ as we estimate σ with s_{n-1}

$$p\text{-value} = 2 \times P\left(t_4 > \frac{3.8 - 0}{\sqrt{\frac{2.58844^2}{5}}}\right)$$

$$= 2 \times P(t_4 > 3.2827)$$

$$= 2 \times 0.015212$$

$$= 0.030423$$

$$< 0.05$$

Hence, we have sufficient evidence to reject H_0 and conclude that the mean difference in scores is non-zero, telling us that the 3rd and 4th round scores are different.

(we conjecture that they are better/lower scoring on their 4th round)

5. X = difference in number of words recalled (where difference = "1 hr" − "24 hr")

assume X to be normally distributed

$X \sim N(\mu, \sigma^2)$

$H_0 : \mu = 5$

$H_1 : \mu \neq 5$

Student	A	B	C	D	E	F	G	H	I	J	K	L
1 hr	14	9	18	12	13	17	16	16	19	8	15	7
24 hr	10	6	14	6	8	10	12	10	14	5	10	5
"1−24"	4	3	4	6	5	7	4	6	5	3	5	2

We assume H_0 to be true

$\alpha = 5\%$

two tailed test

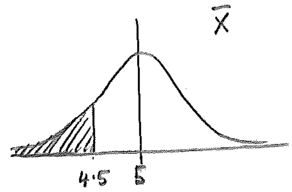
we calculate "1 hr − 24 hr" to give single sample of size $n = 12$

we estimate σ from $s_{n-1} = 1.446$, and as n is small, we perform single sample t-test

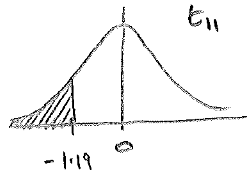
we also know $\bar{x} = 4.5$

so $X \sim N(5, \sigma^2)$

$\bar{X} \sim N(5, \frac{\sigma^2}{12})$ where $\bar{X}$ = mean difference



so $\dfrac{\bar{X} - 5}{\sqrt{\frac{s^2}{12}}} \sim t_{11}$ as we estimate σ with s_{n-1}



so p-value $= 2 \times P\left(t_{11} < \dfrac{4.5 - 5}{\sqrt{\frac{1.446^2}{12}}}\right)$

$= 2 \times P(t_{11} < -1.19782)$

$= 2 \times 0.128079$

$= 0.256159$

> 0.05

Hence we do not have evidence to reject H_0

This suggests that the mean difference is 5 words.

And so the study seems to show that the number of words recalled after 1 hour exceeds that recalled after twenty four hours by a mean of 5 words

6. X = difference in temperature (where difference = satelite - ground)

as each temperature (Ground and Satellite) are normally distributed, we can assume that their differences are also normally distributed.

Site	1	2	3	4	5	6	7	8	9	10	11
Ground	4·6	17·3	12·2	3·6	6·2	14·8	11·4	14·9	9·3	10·4	7·2
Satellite	4·7	19·5	12·5	4·2	6·0	15·4	14·9	17·8	9·7	10·5	7·4
S-G	0·1	2·2	0·3	0·6	-0·2	0·6	3·5	2·9	0·4	0·1	0·2

so $X \sim N(\mu, \sigma^2)$

$H_0: \mu = 0$

$H_1: \mu > 0$

We assume H_0 is true,

$\alpha = 5\%$

one tail test

we calculate 'Satellite - Ground' to give a single sample of $n=11$, for which we estimate σ to be $s_{n-1} = 1.27129$, and so we perform a t-test on this single sample

we also know $\bar{x} = 0.972727$

So $X \sim N(0, \sigma^2)$

$\bar{X} = N\left(0, \frac{\sigma^2}{11}\right)$ where $\bar{X}$ = mean temperature difference

so $\dfrac{\bar{X} - 0}{\sqrt{\frac{S^2}{11}}} \sim t_{10}$ as we estimate σ with s_{n-1}

$$p\text{-value} = P\left(t_{10} > \frac{0.972727 - 0}{\sqrt{\frac{1.27129^2}{11}}}\right)$$

$$= P(t_{10} > 2.53771)$$

$$= 0.014739$$

$$< 0.05$$

Hence we have evidence to reject H_0 and conclude the mean difference in temperature is greater than zero, which means that Satellite sensors give higher readings than ground sensors.