CIMT Further Statistics P34 Ex 2D.

1. X = no. cars passing in a 5 minute interval.

we assume $X \sim Po(\lambda)$

we observe $\hat{\lambda} = 212$.

we approximate X with normal $Y \sim N(\lambda, \lambda)$, allowed as $\lambda > 10$

a) so 95% CI for λ is $\hat{\lambda} \pm z_{0.975} \times \sqrt{\hat{\lambda}}$

$= 212 \pm 1.96 \times \sqrt{212}$

$= (183.462, 240.538)$

$\approx (183.5, 240.5)$ to 1dp)

b) if we instead look at 1 minute, the we are interested in $\frac{1}{5}Y \sim N\left(\frac{\lambda}{5}, \frac{\lambda}{5^2}\right)$

so 95% CI for mean of 1 minute interval is $\frac{\hat{\lambda}}{5} \pm z_{0.975} \times \sqrt{\frac{\hat{\lambda}}{5^2}}$

$= \frac{212}{5} \pm 1.96 \times \sqrt{\frac{212}{25}}$

$= (36.6925, 48.1075)$

$\approx (36.69, 48.11)$ (to 2dp)

c) for 2 minutes, we are interested in $\frac{2}{5}Y \sim N\left(\frac{2\lambda}{5}, \frac{4\lambda}{25}\right)$

so 90% CI for mean of 2 minute interval is $\frac{2\hat{\lambda}}{5} \pm 1.645\sqrt{\frac{4\hat{\lambda}}{25}}$

$= \frac{2}{5} \times 212 \pm 1.645\sqrt{\frac{4}{25} \times 212}$

$= (75.2202, 94.3798)$

$\approx (75.22, 94.38)$ to 2dp.

d) for 60 minutes, we are interested in $12Y \sim N(12 \times 212, 12^2 \times 212)$

so 99% CI for mean of 60 minute interval is $12 \times 212 \pm 2.57583\sqrt{12^2 \times 212}$

$= (2093.94, 2994.06)$

$\approx (2094, 2994)$ (to 4sf)

2.

X = no. times machine needs reset per $\underline{3}$ nights.

$X \sim Po(\lambda)$

so $\hat{\lambda} = 9+5+11 = 25$

so we approximate X with $Y \sim N(\lambda, \lambda)$, which is allowed as $\lambda > 10$

now we instead want CI for 1 night, which is $\frac{1}{3}Y$

and $\frac{1}{3}Y \sim N(\frac{1}{3}\lambda, \frac{1}{9}\lambda)$

so 95% CI for $\frac{1}{3}Y$ is $\frac{1}{3} \times 25 \pm 1.96 \times \sqrt{\frac{25}{9}}$

$= (5.06673, 11.5999)$

$\approx \underline{(5.1, 11.6)}$ to 1dp)

3. X = no. of organisms in 10cc of liquid

$X \sim Po(\lambda)$

$\hat{\lambda} = 35$

a) we approximate X with $Y \sim N(\lambda, \lambda)$ – allowed as $\lambda > 10$

so 90% confidence interval for $\lambda = \hat{\lambda} \pm 1.645\sqrt{\hat{\lambda}}$

$= 35 \pm 1.645\sqrt{35}$

$= (25.2689, 44.7311)$

$\approx \underline{(25.27, 44.73)}$ (to 2dp)

b) as we are after 1cc, we want $\frac{1}{10}Y \sim N(\frac{1}{10}\lambda, \frac{1}{100}\lambda)$

so 95% confidence interval will be $\frac{35}{10} \pm 1.96\sqrt{\frac{35}{100}}$

$= (2.34047, 4.65953)$

$\approx \underline{(2.34, 4.66)}$ to 2dp

c) as we are after 100cc, we want $10Y \sim N(10\lambda, 100\lambda)$

so 99% CI will be $10 \times 35 \pm 2.57583 \times \sqrt{100 \times 35}$

$= (197.612, 502.388)$

$\approx \underline{(197.61, 502.4)}$ to 1dp.

d) if we found 26 in another 10cc, we'd be looking at $35 + 26 = 61$ in 20cc of liquid.

so let X = no. organisms per 20cc

$X \sim Po(\mu)$

$\hat{\mu} = 61$

approx with $Y \sim N(\mu, \mu)$

so 90% CI for 10cc $\Rightarrow \frac{1}{2}Y \sim N(\frac{1}{2}\mu, \frac{1}{4}\mu) \Rightarrow \frac{61}{2} \pm 1.645 \times \sqrt{\frac{61}{4}} = (24.0766, 36.9234)$

$\approx \underline{(24.08, 36.92)}$

95% CI for 1cc $\Rightarrow \frac{1}{20}Y \sim N(\frac{\mu}{20}, \frac{\mu}{400}) \Rightarrow \frac{61}{20} \pm 1.96\sqrt{\frac{61}{400}} = (2.28461, 3.81539)$

$\approx \underline{(2.28, 3.81)}$

99% CI for 100cc $\Rightarrow 5Y \sim N(5\mu, 25\mu) \Rightarrow 61 \times 5 \pm 2.57583\sqrt{61 \times 25} = (204.411, 405.589)$

$\approx \underline{(204.4, 405.6)}$

Note: In all cases, the extra information from the second sample has reduced the size of each confidence interval. ☺