CIMT Further Statistics p165 Ex8.7

2

a) X = percentage of impurity

E(X) = 16.7 and Var(X) = 3.4^2

Samples of size n = 3

We assume that X is distributed normally

$X \sim N(16.7, 3.4^2)$

$\overline{X} \sim N\left(16.7, \frac{3.4^2}{3}\right)$ where $\overline{\mathrm{X}}$ = mean of percentages from sample of size 3

$UCL = 16.7 + 3\sqrt{\frac{3.4^2}{3}} = 22.589$

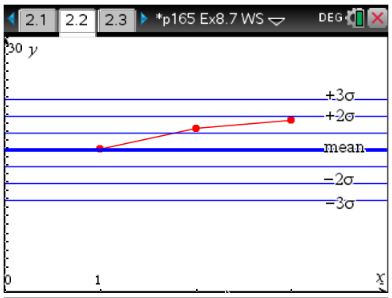
$UWL = 16.7 + 2\sqrt{\frac{3.4^2}{3}} = 20.626$

$LWL = 16.7 - 2\sqrt{\frac{3.4^2}{3}} = 12.774$

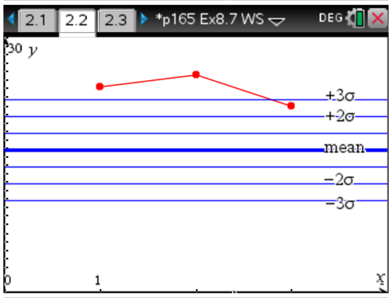
$LCL = 16.7 - 3\sqrt{\frac{3.4^2}{3}} = 10.811$

b) We bear in mind the following Western Electric Company Rules for Control Charts:

- Any single data point falls outside a 3σ limit
- Two out of three consecutive points fall beyond the same 2σ limit
- Four out of five consecutive points fall beyond the same 1σ limit
- Eight consecutive points fall on the same side of the centre line

i) 

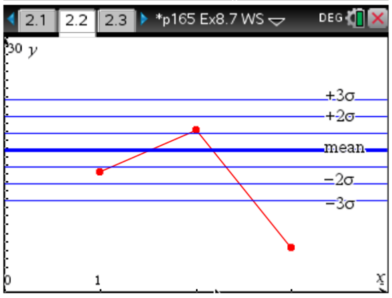
Process in control, no action required.

ii) 

Process out of control.

1st and 2nd values as both beyond +3σ limit

Action required to reduce the mean

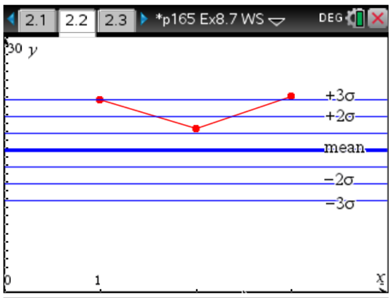
iii) 

Process out of control.

3rd value beyond -3σ limit

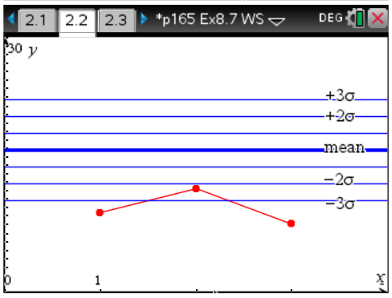
Action require to increase the mean

iv)



Process out of control.
1st and 3rd values are both beyond +3σ limit
Also two out of three values are beyond +2σ limit
Action required to reduce the mean.

v)



Process out of control.
1st and 3rd values are both beyond +3σ limit
Also all three values are beyond +2σ limit
Action required to increase the mean.

5

X = number of lengths needing mended

Sets of 50 lengths were sampled

$X \sim Bin(50, p)$

Approximate X with $Y \sim N(50p, 50pq)$

This is valid if $50p > 5$ and $50q > 5$

$\frac{Y}{50}$ = proportion of lengths needing mended

$\frac{Y}{50} \sim N(p, \frac{pq}{50})$

We estimate p with $\widehat{p} = \frac{17 + 14 + ... + 16 + 12}{11 \times 50} = \frac{174}{550}$

$So\ \frac{Y}{50} \sim N(0.3164, 0.004326)$

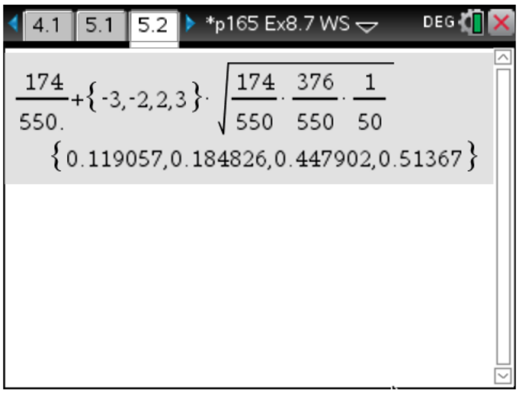
$UCL = 0.3164 + 3 \times \sqrt{0.004326} = 0.5137$

$UWL = 0.3164 + 2 \times \sqrt{0.004326} = 0.4479$

$LWL = 0.3164 - 2 \times \sqrt{0.004326} = 0.1848$

$LCL = 0.3164 - 3 \times \sqrt{0.004326} = 0.1191$

We don’t really need the lower limits, as we are measuring proportions of ‘defectiveness’ and low proportions of this are not concerning to us.



X = number of lines that break under 38N
Sets of 60 lengths were sampled

$X \sim Bin(60, p)$

Approximate X with $Y \sim N(60p, 60pq)$

This is valid if $60p > 5$ and $60q > 5$

$\frac{Y}{60} =$ proportion of lines that break

$\frac{Y}{60} \sim N(p, \frac{pq}{60})$

We estimate p with $\widehat{p} = \frac{14+9+\ldots+6+7}{10 \times 60} = \frac{101}{600}$

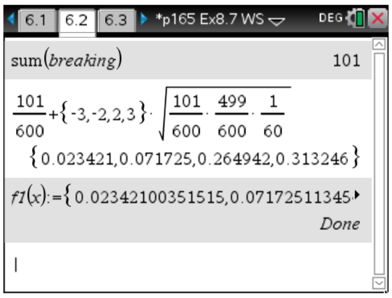
$So\ \frac{Y}{60} \sim N(\frac{101}{600}, \frac{101}{600} \times \frac{499}{600} \times \frac{1}{60})$

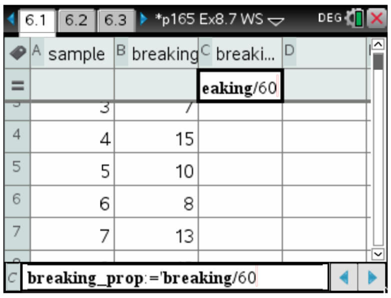
$UCL = \frac{101}{600} + 3 \times \sqrt{\frac{101}{600} \times \frac{499}{600} \times \frac{1}{60}} = 0.3132$

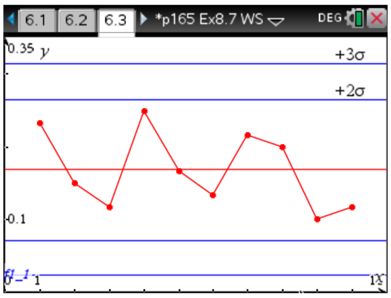
$UWL = \frac{101}{600} + 2 \times \sqrt{\frac{101}{600} \times \frac{499}{600} \times \frac{1}{60}} = 0.2649$

$LWL = \frac{101}{600} - 2 \times \sqrt{\frac{101}{600} \times \frac{499}{600} \times \frac{1}{60}} = 0.0717$

$LCL = \frac{101}{600} - 3 \times \sqrt{\frac{101}{600} \times \frac{499}{600} \times \frac{1}{60}} = 0.0234$







b)
i) if next sample was 18, that's a proportion of 18/60=0.3, which is below the +2σ limit, so no action required.
ii) if next sample was 24, that's a proportion of 24/60=0.4, which is above the +3σ limit, so action required to improve the breaking strains of the line.
iii) if next sample was 1, that's a proportion of 1/60=0.017, which is below the -3σ limit, but as we are measuring defectives, a low number does not concern us. Indeed, it's a good thing, so no action is required.

c) By looking at the proportions of lines that break, rather than the means of the breaking strains, the manufacturer can more clearly see how many lines meet a certain design standard, and thus what they can assure their Angling customers of.
However, by not looking at the mean breaking strain charts, for those that fail the 38N test, they do not know how far off that control criteria they are.