CIMT Further Statistics p65/66 Example

let X_1 = alkalinity of water in upper reaches $\quad X_1 \sim N(\mu_1, 10^2) \quad n_1 = 10$

X_2 = alkalinity of water in lower reaches $\quad X_2 \sim N(\mu_2, 25^2) \quad n_2 = 15$

note - labelling is opposite way around from solution in eBook

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 < \mu_2$

Assume H_0 is true

$\alpha = 1\%$

one tail test

from samples provided $\bar{x}_1 = \dfrac{805}{10} = 80.5$

$$\bar{x}_2 = \frac{1485}{15} = 99$$

we know $X_1 \sim N(\mu_1, 10^2) \qquad X_2 \sim N(\mu_2, 25^2)$

$$\bar{X}_1 \sim N\left(\mu_1, \frac{10^2}{10}\right) \qquad \bar{X}_2 \sim N\left(\mu_2, \frac{25^2}{15}\right)$$

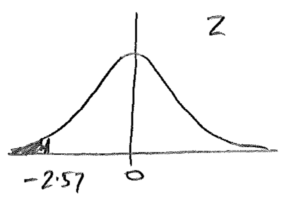
$$\text{so } \bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{10^2}{10} + \frac{25^2}{15}\right)$$

$$\Rightarrow \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{10^2}{10} + \frac{25^2}{15}}} \sim N(0, 1^2)$$

as $\mu_1 = \mu_2$, so $\mu_1 - \mu_2 = 0$

so our test statistic, $z = \dfrac{(80.5 - 99) - (0)}{\sqrt{\frac{10^2}{10} + \frac{25^2}{15}}}$

$$z = -2.57375$$



$P(Z < -2.57) = 0.00503$

< 0.01

Hence we have evidence to reject H_0 and conclude that mean alkalinity of water in the lower reaches is higher than that of the upper reaches.

check using TI-Nspire

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