CIMT Further Stats p94 Ex5A

1.

Score	1	2	3	4
O_i	12	15	19	22
E_i	17	17	17	17

$\Sigma O_i = 68$

H_0 : Scores distributed as a discrete uniform distribution, $U(4)$.

H_1 : Scores are not $U(4)$, and thus die is biased.

Assume H_0 to be true.

$\alpha = 5\%$, one tail test

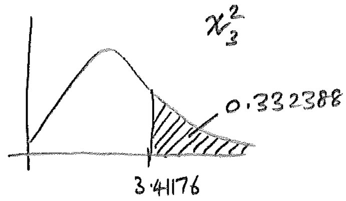
under H_0, E_i are all $\frac{68}{4} = 17$

$$X^2 = \frac{\Sigma (O_i - E_i)^2}{E_i}$$

$$= 3.41176$$

$df = 4 - 1 = 3$.

so $X^2 \sim \chi^2_3$



so $P(X^2 \geq 3.41176) = 0.332388$
> 0.05

Hence we are not in critical region

So we do not have evidence to reject H_0 and so the die seems fair, as they follow a $U(4)$ distribution

2.

Score	1	2	3	4	5	6
f_o	17	20	29	20	18	16
f_e	20	20	20	20	20	20.

H_0: scores are distributed as discrete $U(6)$

H_1: scores are not $\sim U(6)$

Assume H_0 to be true

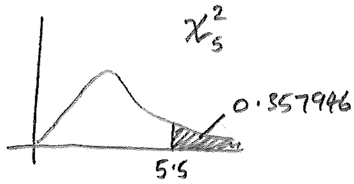
$\alpha = 5\%$, one-tailed test

$df = 5$

$$X^2 = \frac{\sum (O_i - E_i)^2}{E_i}$$

$$= 5.5$$

now $X^2 \sim \chi^2_5$



$P(X^2 \geq 5.5) = 0.357946$

> 0.05

Hence we do not have evidence to reject H_0 and the die appears fair, as they follow a $U(6)$ distribution.

3.

Day	M	Tu	Wed	Thr	Fri
f_o	125	88	85	94	108
f_e	100	100	100	100	100

H_0: absences uniformly distributed, with discrete $U(5)$

H_1: absences dependent on day of week, and not distributed $U(5)$

Assume H_0 to be true

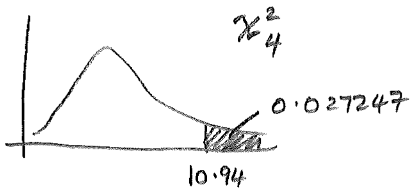
$\alpha = 5\%$, one-tailed test

under H_0, $f_e = 100$, $df = 4$

$$X^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

$$= 10.94$$

$$X^2 \sim \chi^2_4$$



$P(X^2 > 10.94) = 0.027247$

< 0.05

Hence we do have sufficient evidence to reject H_0 and conclude that absences appear dependent on the day of the week.

4.

Day	M	Tu	W	Th	F	Sa	Su.
f_o	60	54	48	53	53	75	77.
f_e	60	60	60	60	60	60	60

$\Sigma f_o = 420$.

H_0: car accidents are uniformally distributed discretely as $u(7)$

H_1: car accidents are not uniformally distributed as $u(7)$

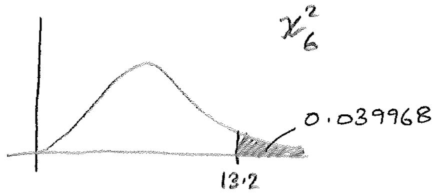
Assume H_0 to be true.

$\alpha = 5\%$ one-tailed test

under H_0, $f_e = 60$ $\quad$ df, $\nu = 7-1 = 6$

$$X^2 = \Sigma \frac{(f_o - f_e)^2}{f_e}$$

$$= 13 \cdot 2$$



$P(X^2 > 13 \cdot 2) = 0.039968$

< 0.05

Hence we have evidence to reject H_0 and conclude that the data are not a sample from a uniform distribution.

5.

Doors	N	S	E	W
f_o	327	402	351	380
f_e	365	365	365	365

$\Sigma f_o = 1460$.

H_0: customers are distributed as discrete $U(4)$

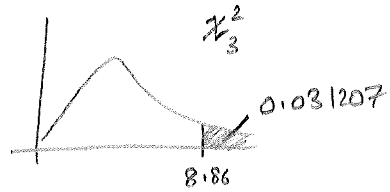
H_1: customers are not distributed as $U(4)$

Assume H_0 to be true.

under H_0, $f_e = 365$

$\alpha = 5\%$, one tailed test.

$df = 4 - 1 = 3$.



$X^2 = \Sigma \frac{(f_o - f_e)^2}{f_e}$

$= 8.86027$

so $P(X^2 > 8.86) = 0.031207$

< 0.05

Hence we have evidence to reject H_0 and conclude that the four exits are not equally used. (It would appear as though most traffic uses the South and West doors).

6.

Blood Type	O	A	B	AB
f_o	87	59	20	4
p_e	0.49	0.38	0.09	0.04
f_e	83.3	64.6	15.3	6.8

$\times 170$

$\Sigma f_o = 170.$

expected ratios $\underbrace{49 : 38 : 9 : 4}_{=100}$

H_0: blood type proportions are similar to general population

H_1: blood type proportions are different to general population

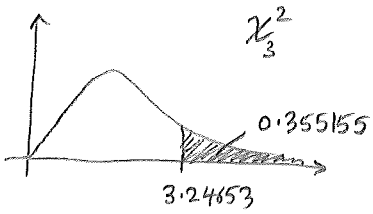
Assume H_0 to be true.

$\alpha = 5\%$, one-tail test

$df = 4 - 1 = 3.$

$$X^2 = \Sigma \frac{(f_o - f_e)^2}{f_e}$$

$$= 3.24653$$



so $P(X^2 > 3.246553) = 0.355155$

> 0.05, so we do not reject H_0.

Hence we do not have evidence to suggest that the small community has different proportions of blood type to those in the general population.