CIMT Further Statistics p70 example

X_1 = sales when windows open

$X_1 \sim N(\mu_1, \sigma_1^2)$

$n_1 = 10$

X_2 = sales when windows closed

$X_2 \sim N(\mu_2, \sigma_2^2)$

$n_2 = 10$.

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 > \mu_2$ (he sells more when windows open)

We assume H_0 to be true

$\alpha = 5\%$

one-tail test

we assume that $\sigma_1 = \sigma_2 = \sigma$, so all the data can be used to estimate σ.

we know that $\bar{x}_1 = 202.18$, $\bar{x}_2 = 188.47$,
$s_1 = 10.7577$, $s_2 = 12.5161$

so $X_1 \sim N(\mu_1, \sigma^2)$ $\quad$ $X_2 \sim N(\mu_2, \sigma^2)$

$\bar{X}_1 \sim N(\mu, \frac{\sigma^2}{10})$ $\quad$ $\bar{X}_2 \sim N(\mu_2, \frac{\sigma^2}{10})$

$$\bar{X}_1 - \bar{X}_2 \sim N\left(\mu_1 - \mu_2, \frac{\sigma^2}{10} + \frac{\sigma^2}{10}\right)$$

$$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma^2}{10} + \frac{\sigma^2}{10}}} \sim N(0, 1^2)$$

$$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sigma\sqrt{\frac{1}{10} + \frac{1}{10}}} \sim N(0, 1^2)$$

$$\frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{s\sqrt{\frac{1}{10} + \frac{1}{10}}} \sim t_{18}$$

where $18 = (n_1 - 1) + (n_2 - 1)$

and s is the estimate of the pooled standard deviations.

$$s^2 = \frac{(10-1)s_1^2 + (10-1)s_2^2}{18}$$

$s = 11.6701$

$$\text{so } t = \frac{202.18 - 188.47 - (0)}{11.6701\sqrt{\frac{1}{10} + \frac{1}{10}}}$$

$t = 2.62693$

p-value $= P(t_{18} > 2.62693)$

$= 0.008551$

< 0.05

So we have evidence to reject H_0 and conclude that the mean sales are greater when the windows are open.