CIMT Further Statistics - p84 - Ex4F

1.

Region	A	B	C	D	E	F	G	H	I	J	K
Before	2·4	2·6	3·9	2·0	3·2	2·2	3·3	2·1	3·1	2·2	2·8
After	3·0	2·5	4·0	4·1	4·8	2·0	3·4	4·0	3·3	4·2	3·9
After-Before	0·6	-0·1	0·1	2·1	1·6	-0·2	0·1	1·9	0·2	2	1·1
Rank	6	$\boxed{2}$	2	11	8	$\boxed{4·5}$	2	9	4·5	10	7

Assumptions : distibution of differences in sales revenue is symmetrical

Justification : paired data

H_0 : median difference $= 0$, (advert compaign had no effect) (difference = After - Before)

H_1 : median difference > 0 , (advert campaign increased sales)

Assume H_0 to be true.

$\alpha = 5\%$

1-tail test

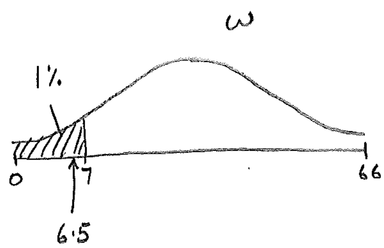
From table above $W_- = 2 + 4·5 = 6·5$

$W_+ = 6+2+11+8+2+9+4·5+10+7 = 59·5$ $\Big\}$ $66 = \frac{1}{2}\times 11\times 12$ ✓check.

and $n = 11$

now $W = \text{minimum}\ (W_-, W_+)$

$= 6·5$



we are interested in $P(W \leq 6·5)$

From tables, for $n=11$, $P(W \leq 13) = 0·05$

$P(W \leq 10) = 0·025$

$P(W \leq 7) = 0·01$

$P(W \leq 5) = 0·005$

so $P(W \leq 6·5)$ is between 0·005 and 0·01

we are in the 5% critical region (we're also in the 1% critical region)

so, we have evidence to reject H_0 at the 5% level and conclude that the median difference is greater than zero. This means that the advertising campaign has increased sales.

2. Assumptions : distribution of differences in clay pigeon shooting scores is symmetrical
Justification : paired data

H_0 : median difference $= 0$ (guns equally good) where difference = CRACKSHOT - FASTFIRE
H_1 : median difference > 0 (crackshot more accurate)

Assume H_0 to be true.
$\alpha = 5\%$ 1 tail test

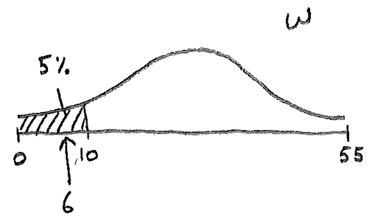
Competitor	A	B	C	D	E	F	G	H	I	J.
Crackshot	93	99	90	86	85	94	87	91	96	79
FastFire	87	91	86	87	78	95	89	84	88	74
C-F	6	8	4	-1	7	-1	-2	7	8	5
rank	6	9.5	4	$\boxed{1.5}$	7.5	$\boxed{1.5}$	$\boxed{3}$	7.5	9.5	5

$n = 10$.

$$\left.\begin{aligned} W_- &= 1.5 + 1.5 + 3 = 6 \\ W_+ &= 6 + 9.5 + 4 + 7.5 + 7.5 + 9.5 + 5 = 49 \end{aligned}\right\} \quad 6 + 49 = 55 = \tfrac{1}{2} \times 10 \times 11 \ \checkmark \text{check}$$

now $W = \min(W_-, W_+)$
$= 6.$

we want $P(W \leq 6)$ for $n = 10$



From tables
$P(W \leq 10) = 0.5$
$P(W \leq 8) = 0.025$
$P(W \leq 5) = 0.01$

So $P(W \leq 6)$ is between 0.01 and 0.025
So at 5%, we are in critical region
Hence, we have evidence to reject H_0 and conclude that the median difference is greater than zero, which means that Crackshot is more accurate than Fastfire.

Ex4F no. 3

Assumptions : distibution of differences in asthmatic index is symmetrical

Justification : paired data

H_0 : median difference $= 0$ (drug has no effect) where difference = PLACEBO − DRUG.

H_1 : median difference > 0 (drug reduces ashmatic index)

Assume H_0 to be true

$\alpha = 5\%$

1 − tail test

Subject	1	2	3	4	5	6	7	8	9	10	11	12
Drug	28	31	17	18	31	12	33	24	18	25	19	17
Placebo	32	33	23	26	34	17	30	24	19	23	21	24
P−D	4	2	6	8	3	5	−3	0	1	−2	2	7
rank	7	3	9	11	5.5	8	$\boxed{5.5}$		1	$\boxed{3}$	3	10

$n = 11$ (as discount the zero)

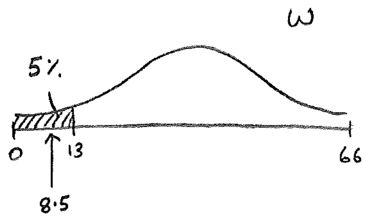
so $W_- = 5.5 + 3 = 8.5$

$W_+ = 7+3+9+11+5.5+8+1+3+10 = 57.5$ } sum to $66 = \frac{1}{2} \times 11 \times 12$ ✓ check

so $W = \min(W_-, W_+)$

$= 8.5$

we want to know $P(W \leq 8.5)$



from tables for $n = 11$, we have

$P(W \leq 13) = 0.05$

$P(W \leq 10) = 0.025$

$P(W \leq 7) = 0.01$

so $P(W \leq 8.5)$ is between 0.025 and 0.01

so we are inside 5% critical region

Hence we have evidence to reject H_0 and conclude that the median difference is greater than zero, which means that the drug appears to reduce the asthmatic index.

p84 Ex4F no. 4

Assumptions : distribution of differences in number of uses of the word "nice" is symmetrical.

Justification: paired data

H_0: median difference $= 0$ (girls can accurately remember) where difference = TRUE - RECORDED

H_1: median difference $\neq 0$ (girls cannot accurately remember)

2-tail test

$\alpha = 5\%$

Assume H_0 to be true

Girl	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O
True	12	20	1	8	0	12	12	17	6	5	24	23	10	18	16
Recorded	9	19	3	14	4	12	16	14	5	9	20	16	11	17	19
T−R	3	1	−2	−6	−4	0	−4	3	1	−4	4	7	−1	1	−3
rank	7	2.5	$\boxed{5}$	$\boxed{13}$	$\boxed{10.5}$		$\boxed{10.5}$	7	2.5	$\boxed{10.5}$	10.5	14	$\boxed{2.5}$	2.5	$\boxed{7}$

so $n = 14$

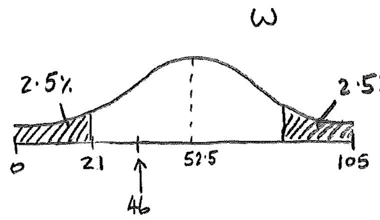
$$\left.\begin{aligned} W_- &= 5+13+10.5+10.5+10.5+2.5+7 = 59 \\ W_+ &= 7+2.5+7+2.5+10.5+14+2.5 = 46 \end{aligned}\right\} 105 = \tfrac{1}{2}\times 14 \times 15 \quad \checkmark \text{ check}$$

so $W = \min(W_-, W_+)$
$= 46$

we are interested in $P(W \leq 46)$ and a 2 tail test

so we either compare $P(W \leq 46)$ to 2.5%

or compare $2 \times P(W \leq 46)$ to 5%



from tables, for $n = 14$,

$P(W \leq 25) = 0.05$

$P(W \leq 21) = 0.025$ etc.

so, it's clear that $P(W \leq 46)$ is more than 0.05

and $2 \times P(W \leq 46)$ is more than 0.10

Hence we are not in either 2.5% tail's critical region

So we do not have evidence to reject H_0

So the median difference is zero

This tells us that girls can accurately remember the frequency with which they use a particular word.

p84 Ex4F no. 5

Assumptions: distribution of differences in the methods is symmetrical

Justification: paired data.

H_0: median difference $=0$ (methods give same results) where difference = METHOD 1 − METHOD 2.
H_1: median difference $\neq 0$ (methods give different results)

Assume H_0 to be true.
2-tail test
$\alpha = 5\%$

Comparative dotplots suggests that method 1 gives slightly higher results than method 2.

Adult	1	2	3	4	5	6	7	8	9	10
Method 1	204	238	209	277	197	226	203	131	282	76
Method 2	199	230	198	253	180	209	213	137	250	82
M1 − M2	5	8	11	24	17	17	−10	−6	32	−6
rank	1	4	6	9	7.5	7.5	$\boxed{5}$	$\boxed{2.5}$	10	$\boxed{2.5}$

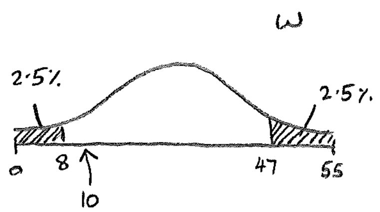
so $n = 10$

$$\left.\begin{aligned} W_- &= 5 + 2.5 + 2.5 = 10 \\ W_+ &= 1+4+6+9+7.5+7.5+10 = 45 \end{aligned}\right\} \quad 55 = \tfrac{1}{2}\times 10 \times 11 \ \checkmark \text{check}$$

so $W = \min(W_-, W_+) = 10$

we want $P(W \leq 10)$ for a 2 tail test

so we either compare $P(W \leq 10)$ to 2.5%
or compare $2P(W \leq 10)$ to 5%



from tables, for $n = 10$,

$P(W \leq 10) = 0.05$
$P(W \leq 8) = 0.025$

so $2P(W \leq 10) = 0.1$ which is more than 5%.

so we are not in the critical tails of the distribution

Hence we do not have evidence to reject H_0, and so we conclude that the median difference is zero.

This tells us that the two methods give the same result.

p84 Ex 4F no. 6

Assumptions: distribution of differences in amount of leather wear is symmetrical

Justification: paired data,

H_0: median difference $= 0$ (wear on boots is the same) where difference $= A - B$.

H_1: median difference $\neq 0$ (wear on boots is different)

Assume H_0 to be true.

2-tail test

$\alpha = 5\%$

Soldier															
A	5.4	2.6	4.3	1.1	3.3	6.6	4.4	3.5	1.2	1.3	4.8	1.2	2.8	2.0	6.1
B	4.7	3.2	3.8	2.3	3.6	7.2	4.4	3.9	1.9	1.2	5.8	2.0	3.7	1.8	6.1
A−B	0.7	−0.6	0.5	−1.2	−0.3	−0.6		−0.4	−0.7	0.1	−1	−0.8	−0.9	0.2	
Rank	8.5	$\boxed{6.5}$	5	$\boxed{13}$	$\boxed{3}$	$\boxed{6.5}$		$\boxed{4}$	$\boxed{8.5}$	1	$\boxed{12}$	$\boxed{10}$	$\boxed{11}$	2	

so $n = 13$

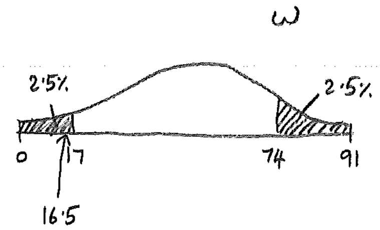
$$\left.\begin{aligned} W_- &= 6.5+13+3+6.5+4+8.5+12+10+11 = 74.5 \\ W_+ &= 8.5+5+1+2 = 16.5 \end{aligned}\right\} 91 = \tfrac{1}{2}\times 13 \times 14 \ \checkmark \text{check.}$$

$W = \min(W_-, W_+) = 16.5$

we want $P(W \leq 16.5)$ for a 2-tail test

so we either compare $P(W \leq 16.5)$ to 2.5%,

or $2P(W \leq 16.5)$ to 5%.



from tables, for $n = 13$,

$P(W \leq 21) = 0.05$

$P(W \leq 17) = 0.025$

$P(W \leq 12) = 0.01$

so $P(W \leq 16.5)$ is between 0.025 and 0.01

so $2P(W \leq 16.5)$ is between 0.05 and 0.02

Hence as $2P(W \leq 16.5)$ is less than 0.05, we are in critical region

So we have evidence to reject H_0 and conclude that the median difference is not zero, which means that the wear on the boots is different.

{ Furthermore, from inspection of the ~~dotplots~~ boxplots, it would appear that Leather A is hardier and wears down less than Leather B }