

CMT Further Statistics - p38 Example

THIS SOLUTION DOES NOT ASSUME PARENT POPULATION STANDARD DEVIATION IS KNOWN.

X = percentage fat content

$$\text{sample mean, } \bar{x} = \frac{228}{12} = 19$$

$$\text{sample st. dev, } s_{n-1} = \sqrt{\frac{4448 - \frac{228^2}{12}}{11}} = \sqrt{10.545} \approx 3.25$$

we shall assume X to be normally distributed

$$X \sim N(\mu, \sigma^2)$$

$$H_0: \mu = 20$$

$$H_1: \mu < 20$$

Assume H_0 to be true

$\alpha = 5\%$ one tail test

$$\text{so } X \sim N(20, \sigma^2)$$

\bar{X} = mean percentage fat content, sample size 12.

$$\bar{X} \sim N(20, \frac{\sigma^2}{12})$$

$$\frac{\bar{X} - 20}{\sqrt{\frac{\sigma^2}{12}}} \sim N(0, 1^2)$$

now we shall estimate σ with $s_{n-1} \Rightarrow$ no longer $N(0, 1^2)$ but rather t_{11} distribution

$$\text{so } \frac{\bar{X} - 20}{\sqrt{\frac{s^2}{12}}} \sim t_{11}$$

$$\text{so test statistic, } t = \frac{\bar{x} - 20}{\sqrt{\frac{3.25^2}{12}}} = \frac{19 - 20}{\sqrt{\frac{3.25^2}{12}}} = -1.06674$$

$$p\text{-value} = P(t_{11} < -1.07)$$

$$= 0.154472$$

$$> 0.05$$

So we have no evidence to reject H_0 and so the mean percentage fat content is 20%. This means that the manufacturer's claim is not supported by this study

