CIMT Further Statistics p72 Ex 4C.

1. X_A = time taken to make starter A

$X_A \sim N(\mu_A, \sigma^2)$

$n_A = 7$

X_B = time taken to make starter B.

$X_B \sim N(\mu_B, \sigma^2)$

$n_B = 7$

$H_0 : \mu_A = \mu_B$

$H_1 : \mu_A \neq \mu_B$.

Assume H_0 to be true

$\alpha = 5\%$

two tail test

we know $\bar{x}_A = 7$ $\quad$ $\bar{x}_B = 6.22857$

$s_A^2 = 0.711805^2$ $\quad$ $s_B^2 = 0.505682^2$

$$\text{pooled } s^2 = \frac{(7-1)0.712^2 + (7-1)0.506^2}{12}$$

$$= 0.617406^2$$

$$\text{test statistic, } t = \frac{(7-6.22857)-(0)}{s\sqrt{\frac{1}{7}+\frac{1}{7}}} \quad \text{where } t \sim t_{12}$$

$$= 2.33754$$

$$\text{p-value} = 2 \times P(t_{12} > 2.33754)$$

$$= 2 \times 0.018777$$

$$= 0.037553$$

$$< 0.05$$

Hence, we have evidence to reject H_0 and conclude that the mean time taken is not the same for both starters.

2. referring to data in Ex4A Qn 1

we shall assume that standard deviation of both samples' populations are equal.

X_A = quantity of dust in boilers A

$n_A = 13$

$X_{A_1} \sim N(\mu_A^2, \sigma^2)$

X_B = quantity of dust in boilers B.

$n_B = 9$

$X_B^2 \sim N(\mu_B^2, \sigma^2)$

$H_0: \mu_A = \mu_B$

$H_1: \mu_A \neq \mu_B$.

Assume H_0 to be true.

$\alpha = 5\%$.

two tailed test

we have $\bar{x}_A = 63.8308$ $\bar{x}_B = 52.8889$

$s_A = 10.6307$ $s_B = 9.00437$

$$\text{pooled } s^2 = \frac{(13-1)\times 10.6307^2 + (9-1)\times 9.00437^2}{12+8}$$

$$= 10.0119^2$$

$$\text{test statistic, } t = \frac{(63.8308 - 52.8889) - (0)}{s\sqrt{\frac{1}{13}+\frac{1}{9}}}$$ has t_{20} distribution

$$= 2.52032$$

$$\text{p-value} = 2 \times P(t_{20} > 2.52032)$$

$$= 2 \times 0.010165$$

$$= 0.020331$$

$$< 0.05$$

Hence we have evidence to reject H_0 and conclude that the mean quantity of dust is not equal between boilers of the two types.

3. referring to data in Ex4A Q12
testing for equality at 1% level.

we will assume that the standard deviations of the two samples' populations are equal

let X_1 = original machine mass filled $\qquad$ X_2 = new machine mass filled

$n_1 = 10$ $\qquad$ $n_2 = 12$

$X_1 \sim N(\mu_1, \sigma^2)$ $\qquad$ $X_2 \sim N(\mu_2, \sigma^2)$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Assume H_0 to be true

$\alpha = 1\%$

two tailed test

we have $\bar{x}_1 = 295.62$ $\qquad$ $\bar{x}_2 = 296.483$

$s_1 = 6.24176$ $\qquad$ $s_2 = 12.8053$

$$\text{pooled } s^2 = \frac{(10-1)\times 6.24176^2 + (12-1)\times 12.8053^2}{9+11}$$

$$= 10.3788^2$$

$$\text{test statistic, } t = \frac{295.62 - 296.483 - (0)}{s\sqrt{\frac{1}{10}+\frac{1}{12}}} \quad \text{which} \sim t_{20}$$

$$= -0.194273$$

$$\text{p-value} = 2\times P(t_{20} < -0.194273)$$

$$= 2 \times 0.423961$$

$$= 0.847922$$

$$> 0.01$$

so we do not have evidence to reject H_0 and so we conclude that the two machine fill the cereal boxes with the same mean mass.

4 referring to Ex4A q№ 3

we shall assume that the standard deviations of the samples' populations are equal

Testing for equal means, at 1% level

X_1 = arm reach measured by worker

$n_1 = 10$

$X_1 \sim N(\mu_1, \sigma^2)$

X_2 = arm reach measured by assistant

$n_2 = 8$

$X_2 \sim N(\mu_2, \sigma^2)$

$H_0 : \mu_1 = \mu_2$

$H_1 : \mu_1 \neq \mu_2$

Assume H_0 to be true

$\alpha = 1\%$

two tail test

we have $\bar{x}_1 = 705.8$ $\quad$ $\bar{x}_2 = 668.125$

$s_1 = 27.704$ $\quad$ $s_2 = 22.7874$

pooled, $s^2 = \dfrac{(10-1) \times 27.704^2 + (8-1) \times 22.7874^2}{9+7}$

$= 25.6691^2$

test statistic, $t = \dfrac{705.8 - 668.125 - (0)}{s\sqrt{\frac{1}{10}+\frac{1}{8}}}$ $\sim t_{16}$

$t = 3.09422$

p-value $= 2 \times P(t_{16} > 3.09422)$

$= 2 \times 0.003482$

$= 0.006964$

< 0.01

Hence, we have evidence to reject H_0 and conclude that the measured functional arm reaches are different from the two members of the research team.

5 referring to EX4A q= 5

test for equality at 5% level

we shall assume that the standard deviations of the samples' proportions are equal

we shall assume that the samples are from normal populations.

X_1 = percentage from photometric method

$X_1 \sim N(\mu_1, \sigma^2)$

$n_1 = 6$

X_2 = percentage from photographic method

$X_2 \sim N(\mu_2, \sigma^2)$

$n_2 = 7$

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$

Assume H_0 to be true

$\alpha = 5\%$, two tailed test

we have $\bar{x}_1 = 86.2333$ $\qquad \bar{x}_2 = 84.7$

$s_1 = 0.804156$ $\qquad s_2 = 1.83303.$

$$\text{pooled } s^2 = \frac{(6-1)\times 0.804156^2 + (7-1)\times 1.83303^2}{6+7-2}$$

$$= 1.45831^2$$

$$\text{test statistic, } t = \frac{86.2333 - 84.7 - (0)}{1.45831\sqrt{\frac{1}{6}+\frac{1}{7}}} \sim t_{11}$$

$$= 1.8899$$

$$\text{p-value} = 2\times P(t_{11} > 1.8899)$$

$$= 2 \times 0.042698$$

$$= 0.085396$$

$$> 0.05$$

So, no evidence to reject H_0 and conclude that the mean percentages of the two measuring techniques are not different.