CIMT Further Stats p31 example

a) i) of 40 jackets, a total of 15 were size 3.

$\Rightarrow$ sample proportion $= \frac{15}{40}$

let X = no. of employees with size 3

$X \sim B(40, \frac{15}{40})$

Approximate to Normal, $Y \sim N\left(40\times\frac{15}{40},\ 40\times\frac{15}{40}\times\frac{25}{40}\right)$ which is valid as $40\times\frac{15}{40} > 5$ and $40\times\frac{25}{40} > 5$. ✓

so $\frac{Y}{40}$ = proportion of employees using size 3

$\frac{Y}{40} \sim N\left(\frac{15}{40},\ \frac{\frac{15}{40}\times\frac{25}{40}}{40}\right)$

$\frac{Y}{40} \sim N\left(\frac{3}{8},\ \frac{3}{512}\right)$

so 95% CI for true proportion p is $\frac{3}{8} \pm 1.95996\sqrt{\frac{3}{512}}$

$= (0.224972, 0.525028)$

$\left(\text{from } \frac{3}{8} + \{-1,1\}\,\text{invNorm}(0.975)\sqrt{\frac{3}{512}}\right)$

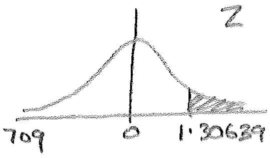
$= (0.225, 0.525)$ to 3dp

ii) the CI is approximate, rather than exact, as we were basing it on a sample estimate for p $\left(\frac{15}{40}\right)$ and we used a normal approximation to the binomial distribution in its generation.

b) i) if we had $\hat{p} \pm 0.1$, then $z\sqrt{\frac{3}{512}} = 0.1$

$z = 0.1\sqrt{\frac{512}{3}}$

$z = 1.30639$



now $P(Z > 1.30639) = 0.095709$

so CI is $1 - 2\times 0.095709$

$= 0.808581$

$= 81\%$ (2sf).

ii) if $\hat{p} \pm 0.1$ was 95% interval, then $0.1 = 1.95996\sqrt{\frac{\frac{15}{40}\times\frac{25}{40}}{n}}$

$0.051021 = \sqrt{\frac{\frac{15}{40}\times\frac{25}{40}}{n}}$

$0.002603 = \frac{\frac{15}{40}\times\frac{25}{40}}{n}$

$n = 90.0342$

So a sample of size 90 would be needed.