CIMT Further Stats p14 Ex 1B.

1. X = amount of soft drink

$X \sim N(475, 20^2)$

$E(X) = 475$

$Var(X) = 20^2$

a) let $T = X_1 + X_2$ where $E(X_i) = 475$

$Var(X_i) = 20^2$

$E(T) = E(X_1 + X_2)$

$= E(X_1) + E(X_2)$

$= 475 + 475$

$= 950$

$Var(T) = Var(X_1 + X_2)$

$= Var(X_1) + Var(X_2)$

$= 20^2 + 20^2$

$= 800$

Sum of nomal distributions is also normal

so $T \sim N(950, 800)$

b) so let C = capacity of cup.

$C \sim N(500, 30^2)$

ie. $E(C) = 500$

$Var(C) = 30^2$

let $D = C - X$

$E(D) = E(C - X)$

$= E(C) - E(X)$

$= 500 - 475$

$= 25$

$Var(D) = Var(C - X)$

$= Var(C) + Var(X)$

$= 20^2 + 30^2$

$= 1300$

difference of nomal distributions is also nomal

$D \sim N(25, 1300)$

2.

X_i = weight of piece of fudge i

$X_i \sim N(34, 5^2)$

$E(X_i) = 34$

$Var(X_i) = 5^2$

a) $B = X_1 + X_2 + \ldots + X_{15}$ B = weight of bag of 15 pieces of fudge

$$E(B) = E(X_1 + X_2 + \ldots + X_{15})$$
$$= E(X_1) + E(X_2) + \ldots + E(X_{15})$$
$$= 15 \times 34$$
$$= 510$$

$$Var(B) = Var(X_1 + X_2 + \ldots + X_{15})$$
$$= Var(X_1) + Var(X_2) + \ldots + Var(X_{15})$$
$$= 15 \times 5^2$$
$$= 375.$$

sum of normal distributions is normal

$B \sim N(510, 375)$

$$P(490 < B < 540) = P\left(\frac{490-510}{\sqrt{375}} < Z < \frac{540-510}{\sqrt{375}}\right)$$ where $Z \sim N(0, 1^2)$

$$= P(-1.0328 < Z < 1.54919)$$

$$= 0.788483\ldots$$ from normcdf $(-1.03, 1.55)$

$$\approx \underline{\underline{0.7885}} \quad (4dp)$$

b) let $B_{15} \sim N(510, 375)$

$B_{16} \sim N(544, 400)$

we want $P(B_{15} > B_{16}) = P(B_{15} - B_{16} > 0)$

let $E = B_{15} - B_{16}$

$E \sim N(510 - 544, 375 + 400)$

$E \sim N(-34, 775)$

so $$P(B_{15} - B_{16} > 0) = P\left(Z > \frac{0-(-34)}{\sqrt{775}}\right)$$

$$= P(Z > 1.22132)$$

$$= 0.110983\ldots$$ from normCdf $(1.22, 9E99)$.

$$\approx \underline{\underline{0.1110}} \quad (4dp)$$

3.

X = time taken for taxi to arrive

$X \sim N(19, 3^2)$

a) i) $P(X<15) = P(Z< \frac{15-19}{3})$

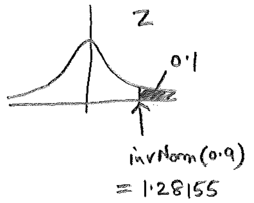
$= P(Z<-\frac{4}{3})$

$= 0.091211...$

≈ 0.0912 (4dp)

ii) $P(X>x) = 0.1$

$P(Z> \frac{x-19}{3}) = 0.1$



$\frac{x-19}{3} = 1.28155$

$x = 19 + 3 \times 1.28155$

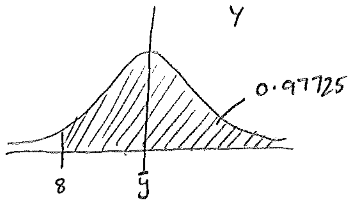
$x = 22.8447$

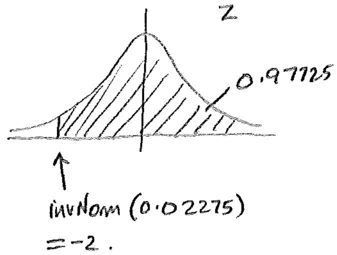
$x \approx 22.8$ mins. (1dp)

b) Y = time taken for Blue Star taxi to arrive

$\therefore Var(Y) = 7^2$

$P(Y>8) = 0.97725$





so $P(Y>8) = 0.97725$

$P(Z> \frac{8-\bar{y}}{7}) = 0.97725$

$\frac{8-\bar{y}}{7} = -2$

$8-\bar{y} = -14$

$\bar{y} = 8+14$

$\bar{y} = 22$ mins

c) $X \sim N(19, 3^2)$

$Y \sim N(22, 7^2)$

let $T = X - Y$

$E(T) = E(X) - E(Y)$
$= 19 - 22$
$= -3$

$Var(T) = Var(X-Y)$
$= Var(X) + Var(Y)$
$= 3^2 + 7^2$
$= 58$

so $T \sim N(-3, 58)$

$P(\text{Toto arrive first}) = P(X<Y)$

$= P(X-Y<0)$

$= P(T<0)$

$= P(Z< \frac{0-(-3)}{\sqrt{58}})$

$= P(Z<0.393919) \approx 0.65318... \approx 0.6532$ (4dp)

3 d) $X \sim N(19, 3^2)$

$Y \sim N(22, 7^2)$

is $P(X<10)$ less than $P(Y<10)$?

$P(X<10)$

$= P\left(Z < \frac{10-19}{3}\right)$

$= P\left(Z < -\frac{9}{3}\right)$

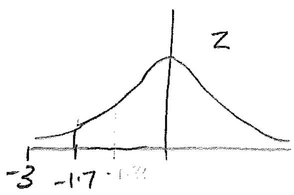
$= P(Z < -3)$

$P(Y<10)$

$= P\left(Z < \frac{10-22}{7}\right)$

$= P\left(Z < -\frac{12}{7}\right)$

$= P(Z < -1.71429\ldots)$



so $P(X<10) < P(Y<10)$

Hence Blue Star are more likely to arrive, as they have the larger probability of arriving within 10 mins, compared to Toto's taxis.

Call Blue Star.